

# Upper Limits on Electric and Weak Dipole Moments of $\tau$ -Lepton and Heavy Quarks from $e^+e^-$ Annihilation

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## Abstract

The total cross-sections measured at LEP for  $e^+e^-$  annihilation into  $\tau^+\tau^-$ ,  $c\bar{c}$  and  $b\bar{b}$  at  $2E \simeq 200$  GeV are used to derive the upper limits  $3 \cdot 10^{-17}$ ,  $5 \cdot 10^{-17}$ ,  $2 \cdot 10^{-17}$  e-cm for the electric dipole moments and  $4 \cdot 10^{-17}$ ,  $7 \cdot 10^{-17}$ ,  $2.5 \cdot 10^{-17}$  e-cm for the weak dipole moments of the  $\tau$ -lepton,  $c$ -, and  $b$ -quarks, respectively. Some of the existing limits on these moments are improved and for the  $b$ -quark the improvement is rather significant.

## 1. Theory

The existence of the electric dipole moment (EDM)  $d$  and weak dipole moment (WDM)  $d^w$  would imply  $CP$  violation. Since the expected values of  $d$  and  $d^w$  in the Standard Model (SM) are extremely small the measurement of significantly larger values would be evidence for physics beyond the SM.

In this paper we consider the reaction of the high-energy electron-positron annihilation into fermions supposing that the final particles have the EDM  $d$  and WDM  $d^w$  (see the corresponding Feynman graphs at tree level in Fig. 1).

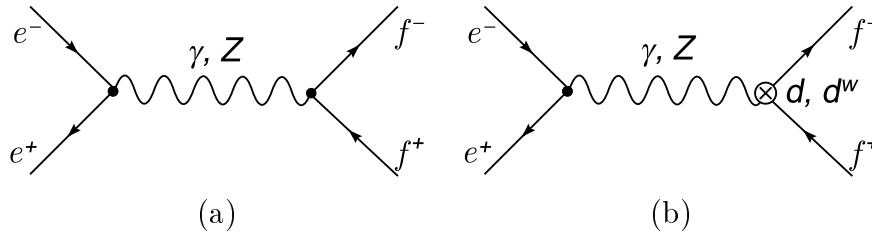


Figure 1: (a) the diagrams with the regular vertices;  
(b) the diagrams with the dipole moments vertices

The final fermions could be both leptons and quarks (if the quark-antiquark pair production occurs far above the threshold and interaction between them may be neglected).

The effective Lagrangian describing interaction of the EDM with the electromagnetic field and the WDM with the  $Z$ -boson field is

$$L_{eff} = -\frac{1}{2}\bar{\psi}\gamma_5\sigma^{\mu\nu}\psi(dF_{\mu\nu} + d^w F_{\mu\nu}^Z), \quad (1)$$

where  $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$ ,  $\sigma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$ .

The EDM and WDM are supposed to have no imaginary parts, since the latter would violate not only  $CPT$  invariance, but also Hermitian character of the Lagrangian.

Fermions in the reaction  $e^+e^- \rightarrow f\bar{f}$  are produced in triplet states if the production mechanism is regular:  $^3S_1$  and  $^3D_1$  if the vertex is a vector  $\gamma_\mu$ , or  $^3P_1$  if the vertex is an axial one  $\gamma_5\gamma_\mu$ . However, if produced via the  $CP$ -odd electric or weak dipole moment vertex  $d\gamma_5\sigma_{\mu\nu}q_\nu$  or  $d^w\gamma_5\sigma_{\mu\nu}q_\nu$ , the fermions will be in the singlet state  $^1P_1$ . Therefore, if polarization of the final particles is not taken into account the dipole moment vertices do not interfere with the regular ones, and hence their contribution to the cross-section is of second order in the dipole moments.

Considering the reaction in the centre-of-mass system assuming unpolarized electron and positron beams, neglecting the electron mass and summing over polarization of the final particles we obtain the following expressions of the squared matrix elements  $M_d^2$ ,  $M_{d^w}^2$  and the interference term  $M_{dd^w}^2$ :

$$M_d^2 = 4e^2 d^2 E^2 \left(1 - \frac{m_f^2}{E^2}\right) \sin^2 \theta, \quad (2)$$

$$M_{d^w}^2 = \frac{16e^2 (d^w)^2}{\sin^2 \theta_W \cos^2 \theta_W} \frac{E^4 (E^2 - m_f^2)}{(4E^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} (V_e^2 + A_e^2) \sin^2 \theta, \quad (3)$$

$$M_{dd^w}^2 = -\frac{16e^2 dd^w}{\sin \theta_W \cos \theta_W} \frac{E^2 (E^2 - m_f^2) (4E^2 - m_Z^2)}{(4E^2 - m_Z^2)^2 + m_Z^2 \Gamma_Z^2} V_e \sin^2 \theta, \quad (4)$$

where  $E$  is the beam energy,  $V_e = -\frac{1}{2} + 2 \sin^2 \theta_W$ ,  $A_e = -\frac{1}{2}$ .

The total cross-section of the process  $e^+e^- \rightarrow f\bar{f}$  looks as follows:

$$\sigma = \sigma_{SM} + \frac{1}{32\pi} \frac{1}{4E^2} \sqrt{1 - \frac{m_f^2}{E^2}} N_c \int (M_d^2 + M_{d^w}^2 + M_{dd^w}^2) d(\cos \theta), \quad (5)$$

where  $N_c = 1$  if the final particles are leptons and  $N_c = 3$  for quarks.

The interference term  $M_{dd^w}^2$  is numerically suppressed due to smallness of  $V_e$ .

## 2. Analysis of experimental data

Since the relative contribution of the  $M_d^2$  and  $M_{d^w}^2$  terms grows with energy, the best limits can be obtained from the highest energy experiments. Therefore, we use the LEP-II measurements of the two-fermion final states combined by LEPWWG [1] in order to set the limits. The integrated luminosity amounts to about  $670 \text{ pb}^{-1}$  per experiment collected at centre-of-mass energies between 183 and 207 GeV.

The measured values and the SM predictions of  $e^+e^- \rightarrow \tau^+\tau^-$  cross-sections were obtained from Table 3.2 of Ref. [1]. The measured values and the SM predictions of  $e^+e^- \rightarrow c\bar{c}$  and  $e^+e^- \rightarrow b\bar{b}$  cross-sections were obtained as a combination of  $e^+e^- \rightarrow q\bar{q}$  ones (Table 3.2 of Ref. [1]) and  $R_c$ - and  $R_b$ -fractions of the  $c\bar{c}$  and  $b\bar{b}$  among the  $q\bar{q}$  final states (Tables 3.9, 3.10 of Ref. [1]). The errors of the  $c\bar{c}$  and  $b\bar{b}$  cross-sections are dominated by the uncertainties of  $R_c$  and  $R_b$ , respectively.

The measured cross sections coincide within errors with the SM predictions. The  $\chi^2$  fits of residuals by the second term in Formula (5) with non-zero  $d$ - or  $d_w$ -terms yield the 95% CL limits  $3 \cdot 10^{-17}$ ,  $5 \cdot 10^{-17}$ ,  $2 \cdot 10^{-17} \text{ e}\cdot\text{cm}$  for the electric and  $4 \cdot 10^{-17}$ ,  $7 \cdot 10^{-17}$ ,  $2.5 \cdot 10^{-17} \text{ e}\cdot\text{cm}$  for the weak dipole moments of the  $\tau$ -lepton,  $c$ -, and  $b$ -quarks, respectively.

Table 1: Comparison of our results with the previous ones

Previous limits (e·cm)	Our limits (e·cm)
$d_\tau < 1.4 \cdot 10^{-16}$ [2], $-2.2 < \text{Re}(d_\tau) < 4.5$ ( $10^{-17}$ ) and $-2.5 < \text{Im}(d_\tau) < 0.8$ ( $10^{-17}$ ) [3], $d_\tau < 1.1 \cdot 10^{-17}$ [4]	$d_\tau < 3 \cdot 10^{-17}$
$d_c < 8.9 \cdot 10^{-17}$ [6]	$d_c < 5 \cdot 10^{-17}$
$d_b < 8.9 \cdot 10^{-17}$ [6]	$d_b < 2 \cdot 10^{-17}$
$ \text{Re}(d_\tau^w)  < 5.0 \cdot 10^{-18}$ and $ \text{Im}(d_\tau^w)  < 1.1 \cdot 10^{-17}$ [7], $-3.56 < \text{Re}(d_\tau^w) < 2.26$ ( $10^{-18}$ ) and $-0.69 < \text{Im}(d_\tau^w) < 0.77$ ( $10^{-17}$ ) [8]	$d_\tau^w < 4 \cdot 10^{-17}$
$d_c^w < 5.7 \cdot 10^{-17}$ [9]	$d_c^w < 7 \cdot 10^{-17}$
$d_b^w < 6.0 \cdot 10^{-16}$ [9]	$d_b^w < 2.5 \cdot 10^{-17}$

### 3. Discussion

Now we briefly compare our results with other ones. For the  $\tau$ -lepton the upper limit  $d_\tau < 1.4 \cdot 10^{-16}$  e·cm was obtained from the analysis of angular distribution of the tau pairs in the  $e^+e^- \rightarrow \tau^+\tau^-$  reaction at  $2E \simeq 35$  GeV [2]. The best direct experimental restrictions on the  $\tau$  EDM are  $-2.2 < \text{Re}(d_\tau) < 4.5$  ( $10^{-17}$  e·cm) and  $-2.5 < \text{Im}(d_\tau) < 0.8$  ( $10^{-17}$  e·cm) [3]. The result was obtained in the  $e^+e^- \rightarrow \tau^+\tau^-$  reaction at  $2E \simeq 10$  GeV using a technique which takes into account  $\tau$  polarization. The even better upper limit  $d_\tau < 1.1 \cdot 10^{-17}$  e·cm was obtained from the partial width  $\Gamma(Z \rightarrow \tau^+\tau^-)$  measured at LEP in Z peak [4]. However, a model dependent relationship between the weak and electric dipole moments is used. Quite recently upper limit  $d_\tau < (1 \pm 1) \cdot 10^{-16}$  e·cm at momenta about  $m_\tau \sim 1 - 2$  GeV have been derived from the precision measurements of the electron EDM [5].

For the  $c$ - and  $b$ -quarks the limits  $d_c < 8.9 \cdot 10^{-17}$  e·cm and  $d_b < 8.9 \cdot 10^{-17}$  e·cm were obtained from measurements at the Z peak [6] with model assumptions similar to Ref. [4].

For the  $\tau$ -lepton the upper limits  $|\text{Re}(d_\tau^w)| < 5.0 \cdot 10^{-18}$  e·cm,  $|\text{Im}(d_\tau^w)| < 1.1 \cdot 10^{-17}$  e·cm were obtained from the reaction  $e^+e^- \rightarrow \tau^+\tau^-$  at energies near the Z resonance [7]. The results combining ALEPH, DELPHI and OPAL data lead to  $-3.56 < \text{Re}(d_\tau^w) < 2.26$  ( $10^{-18}$  e·cm) and  $-0.69 < \text{Im}(d_\tau^w) < 0.77$  ( $10^{-17}$  e·cm) [8].

The bounds on  $d_c^w$  and  $d_b^w$  were obtained from the decays  $Z \rightarrow c\bar{c}$ ,  $Z \rightarrow b\bar{b}$  and they are  $d_c^w < 5.7 \cdot 10^{-17}$  e·cm,  $d_b^w < 6.0 \cdot 10^{-16}$  e·cm [9].

Thus, the upper limits obtained from the contribution of the EDM to the total cross section of  $f\bar{f}$ -production at LEP-II energy are similar to the best direct experimental limits for  $d_\tau$  and are somewhat tighter for  $d_c$  and  $d_b$ .

Our upper limit on the  $\tau$  WDM is worse than ones in Refs. [7, 8] because the analysis therein used a technique which takes  $\tau$  polarization into account. For the  $c$  quark our bound on the WDM is similar to and for the  $b$  is better than ones in Ref. [9].

For convenience we present a comparison of our results with the previous ones in Table 1.

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